

Towards a controlled study of the QCD critical point

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Abstract. The phase diagram of QCD, as a function of temperature T and quark chemical potential μ , may contain a critical point (μ_E, T_E) whose non-perturbative nature makes it a natural object of lattice studies. However, the sign problem prevents the application of standard Monte Carlo techniques at non-zero baryon density. We have been pursuing an approach free of the sign problem, where the chemical potential is taken as imaginary and the results are Taylor-expanded in μ/T about $\mu = 0$, then analytically continued to real μ .

Within this approach we have determined the sensitivity of the critical chemical potential μ_E to the quark mass, $d(\mu_E)^2/dm_q|_{\mu_E=0}$. Our study indicates that the critical point moves to *smaller* chemical potential as the quark mass *increases*. This finding, contrary to common wisdom, implies that the deconfinement crossover, which takes place in QCD at $\mu = 0$ when the temperature is raised, will remain a crossover in the μ -region where our Taylor expansion can be trusted. If this result, obtained on a coarse lattice, is confirmed by simulations on finer lattices now in progress, then we predict that no *chiral* critical point will be found for $\mu_B \lesssim 500$ MeV, unless the phase diagram contains additional transitions.

1. Motivation

QCD has a rich phase diagram as a function of quark chemical potential μ and temperature T , with at least 3 regimes: confining (low μ, T), quark-gluon plasma (high T) and color superconducting (high μ). Lattice simulations have provided clear evidence that, for $\mu=0$, the temperature-driven "transition" between the first 2 regimes is actually a crossover [1]. Then, the commonly expected (μ, T) phase diagram, depicted Fig. 1 (left), contains a critical point caused by the $\mu=0$ crossover turning into a first-order transition. This critical point is the object of both experimental search and theoretical lattice investigation. If located at small enough chemical potential, it could be discovered in high-energy, not-so-heavy ion collisions at RHIC or LHC. In fact, it has been predicted to lie at $(\mu_E, T_E) = (120(13), 162(2))$ MeV in a celebrated lattice study [2].

However, the numerical method of Ref. [2], which consists of reweighting results obtained from $\mu=0$ simulations to $\mu \neq 0$, is known to fail and give wrong results without warning (see, e.g., [3]) when the overlap between the $\mu=0$ Monte Carlo ensemble and the target $\mu \neq 0$ ensemble becomes insufficient. Therefore, the abrupt change observed in [2] as a function of μ (see Fig. 1 there) might be caused by an abrupt breakdown of the numerical approach. In addition the needed reweighting factors to the $\mu \neq 0$ target

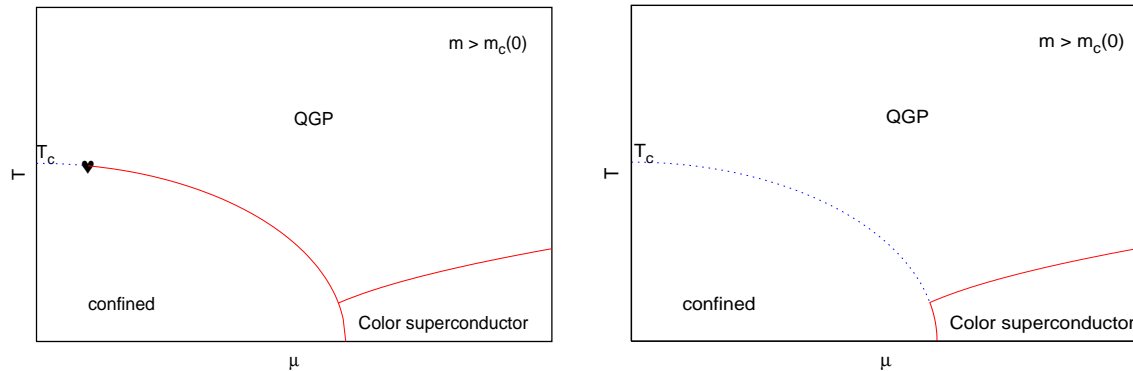


Figure 1. Conventional (left) and simplest possible unconventional (right) phase diagram of QCD as a function of chemical potential μ and temperature T . Dotted lines denote crossovers, solid lines first-order phase transitions. The QCD critical point (end of first-order line), marked by a \heartsuit on the left, is absent on the right.

ensemble are not always positive, because of the notorious “sign problem” affecting the $\mu \neq 0$ determinant. This makes the statistical errors more difficult to control. Finally, the lattices used in [2] have $N_t = 4$ time-slices, corresponding to a very coarse lattice spacing $a \sim 0.3$ fm. As acknowledged in [2] already, the results may change considerably after extrapolation to the continuum limit $a \rightarrow 0$. Therefore, the landmark result of [2] should not be considered the final word on the QCD critical point. A more cautious crosscheck like the one we present here is warranted.

On the theoretical side, the expectation of a critical point as in Fig. 1 (left) can be traced back to two assumptions: (i) QCD with $N_f=2$ massless flavours, at $\mu=0$, has a second-order temperature-driven phase transition in the $O(4)$ universality class [4]. This implies, for this theory, the existence of a tricritical point at $(\hat{\mu}, \hat{T})$, and for the $N_f=2+1$ theory that of another tricritical point at $(\mu=0, T^*, m_{u,d}=0, m_s^*)$. (ii) These two tricritical points are analytically connected by a line in the $(\mu, T, m_{u,d}=0, m_s)$ space. However, assumption (i) depends on the strength of the axial $U_A(1)$ symmetry breaking at T_c . And assumption (ii) has no other basis than simplicity. This motivates us to take a careful look at the phase diagram of $N_f=2+1$ QCD, generalized to arbitrary quark masses $m_u=m_d=m_{u,d}$ and m_s . We present our $\mu=0$, then $\mu \neq 0$ findings below.

2. Approach

To protect ourselves against the pitfalls of reweighting to $\mu \neq 0$, we consider the effect of an *imaginary* chemical potential $\mu = i\mu_I$ [7, 6]. Then the fermion determinant is positive and no overlap problem occurs because we do not reweight. The drawback is that, in order to analytically continue an observable, its μ -dependence is fitted to low-order Taylor series about $\mu=0$, thus introducing a truncation error. As a safeguard, we also calculate the coefficients without truncations [9]. Note that these derivatives can be expressed, and measured, as non-local observables in the $\mu=0$ ensemble. This is the strategy pursued successfully in [8] for the pressure. Our approach is computationally

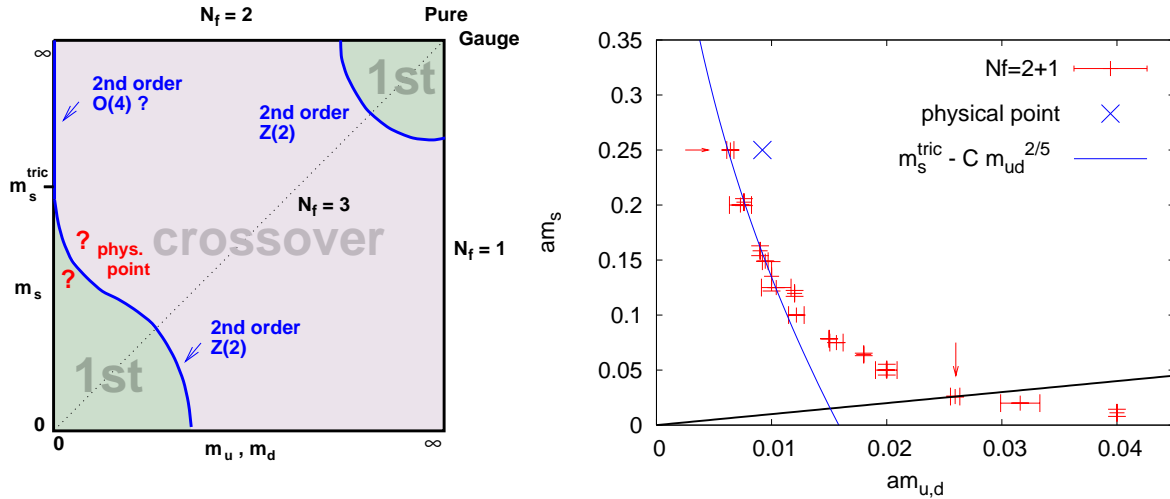


Figure 2. Order of the temperature-driven phase transition (at $\mu=0$), as a function of the light quark mass $m_{u,d}$ and the strange quark mass m_s . On the left, theoretical expectations [5]. On the right, $N_t = 4$ lattice results for light quarks, from [6]. The straight line corresponds to $N_f = 3$ ($m_s = m_{u,d}$). The curve is the expected scaling behaviour from a tricritical point at $m_{u,d} = 0, m_s \sim 500$ MeV. The physical point, marked by a \times , is in the crossover region.

more economical, and we extend it to the critical line.

The first step is to determine, at $\mu=0$, the line in the $(m_{u,d}, m_s)$ plane for which the temperature-driven phase transition is second order. For lighter quarks, the transition becomes first order as in the chiral limit; for heavier quarks, lattice simulations show a crossover. Theoretical expectations are displayed in Fig. 2 (left). Our determination of the critical line on an $N_t = 4$ ($a \sim 0.3$ fm) lattice is shown Fig. 2 (right), and matches expectations, including the possible scaling behaviour in the vicinity of a tricritical point at $(m_{u,d}=0, m_s^{\text{tric}} \sim 500$ MeV). Also, the physical point (with real-world π and K meson masses) lies, as expected, in the crossover region.

We now choose a point on the critical line just determined, and monitor the effect of a small μ . This exercise has been performed, with similar results, at the two points marked by arrows in Fig. 2 (right), with higher accuracy for the $N_f = 3$ case. If conventional wisdom is right, then the surface spanned by the critical line as μ is turned on will bend towards the crossover region, so as to produce a critical point at some small μ value for physical quark masses. This is depicted Fig. 3 (left). On the contrary, an opposite curvature will indicate the absence of a critical point at small μ , where our truncated Taylor expansion is a good approximation. This is illustrated Fig. 3 (right). The issue is thus to determine whether the $\mu=0$ second order transition becomes first order or crossover for a small μ . We have now confirmed our first study [6] by a second one using higher statistics and an improved numerical method [9]. We measure the change in the Binder cumulant $B_4(X) \equiv \langle (X - \langle X \rangle)^4 \rangle / \langle (X - \langle X \rangle)^2 \rangle^2$, with $X = \bar{\psi}\psi$. At the second order transition, B_4 takes the value 1.604 dictated by the 3d Ising universality class. Reweighting with a small imaginary μ induces small changes in B_4 , which can be measured accurately because statistical errors largely cancel between the reweighted

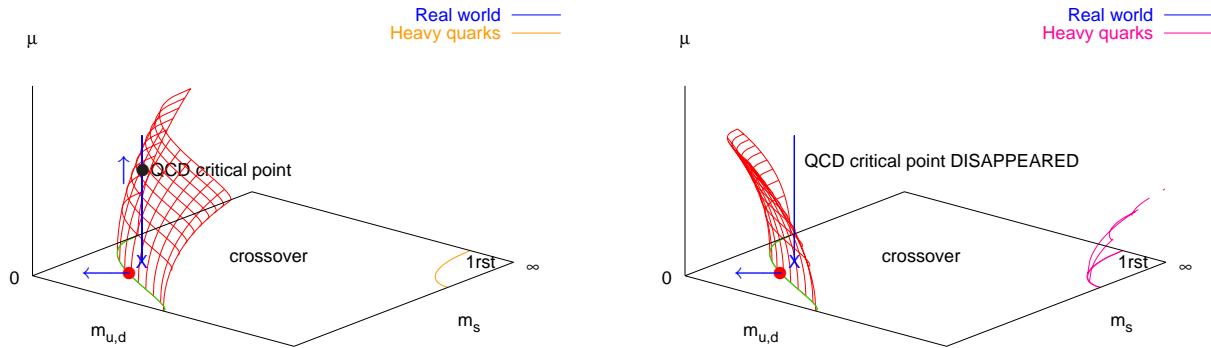


Figure 3. Critical surface spanned by the critical line of Fig. 2 as μ is turned on. Depending on the curvature of this surface, a QCD critical point is present (left) or absent (right). For heavy quarks, the curvature (right) has been determined in [10]. The arrows indicate the effect of a finer lattice: the distance between the physical quark masses and the critical surface increases, driving the critical point (left) to larger μ .

and original ensembles. Our current result for the $N_f=3$ ($m_{u,d}=m_s$) theory is

$$\frac{m_c(\mu_q)}{m_c(0)} = 1 - 3.3(5) \left(\frac{\mu_q}{\pi T} \right)^2 - 12(6) \left(\frac{\mu_q}{\pi T} \right)^4 + \dots \quad (1)$$

supporting the unconventional Fig. 3 (right), both in the leading and subleading term.

It is now mandatory to repeat this study on a finer lattice, in order to gain some control over the continuum limit. $N_t=6$ ($a \sim 0.2$ fm) simulations are in progress. The first step, at $\mu=0$, already reveals an important shift of the critical line towards the origin: for the $N_f=3$ theory, the pion mass, measured at $T=0$ with the critical quark mass, decreases from $1.6 T_c$ to $0.95 T_c$ [9]. As shown Fig. 3, this considerably increases the distance of the critical surface to the physical point, pushing the critical point to larger μ (left), or requiring larger higher-order terms of the right sign to bend the critical surface “back” (right). Regardless of the sign of the curvature in the continuum limit, this trend alone makes a QCD chiral critical point at small $\mu_q/T \lesssim 1$ unlikely.

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